## The University of Texas at Austin Dept. of Electrical and Computer Engineering Midterm #2

Date: November 10, 2005 Course: EE 313 Evans

Name:			
	Last.	First	

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise.

Problem	Point Value	Your score	Topic
1	20		Differential Equation
2	20		Discrete-Time Stability
3	20		Discrete-Time Tapped Delay Line
4	24		Analog Filter Design
5	16		Potpourri
Total	100		

## **Problem 1.1** Differential Equation. 20 points.

For a continuous-time linear time-invariant (LTI) system with input x(t) and output y(t) governed by the differential equation

$$\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = x(t)$$

for  $t \ge 0^-$ .

(a) What is the transfer function? 6 points.

(b) What are the values of the poles and zeroes of the transfer function. 6 points.

(c) Give a formula for the step response of the system. 8 points.

**Problem 1.2** Discrete-Time Stability. 20 points.

In this problem, a discrete-time system has an input signal denoted by x[n] and an output signal denoted by y[n].

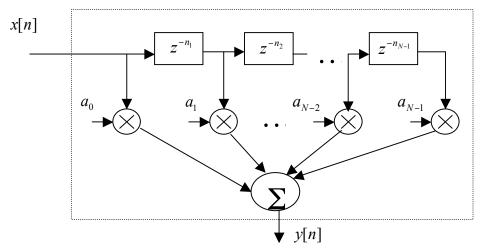
(a) Is the system defined by y[n+2]-4y[n]=x[n+2] asymptotically stable, marginally stable, or unstable? Why? 8 points.

(b) Let K be a real-value constant. For what values of K is the following system asymptotically stable? y[n+2] - Ky[n] = x[n+2]. Why? 8 points.

(c) Rewrite y[n+2]-4y[n] = x[n+2] in causal form. Assuming that the system is linear and time-invariant, compute the first three samples of the impulse response. 4 points

## **Problem 1.3** Discrete-Time Tapped Delay Line. 20 points.

A linear time-invariant (LTI) discrete-time tapped delay line with input x[n], output y[n], and N-1 delay elements is shown below as a block diagram. The notation  $z^{-m}$  means a delay of m samples. Each delay element has a different amount of delay.



- (a) Give a formula for the input-output relationship. 8 points.
- (b) Plot by hand the impulse response h[n]. 4 points.

(c) Plot by hand the step response. 4 points.

(c) Give a formula for the system time constant. 4 points.

## **Problem 1.4** Analog Filter Design. 24 points.

A second-order filter has two poles, and 0, 1, or 2 zeros. The transfer function for a second-order analog filter with poles  $p_0$  and  $p_1$  and zeros  $z_0$  and  $z_1$  follows:

$$H(s) = b_0 \frac{(s - z_0)(s - z_1)}{(s - p_0)(s - p_1)} = b_0 \frac{s^2 - (z_0 + z_1)s + z_0 z_1}{s^2 - (p_0 + p_1)s + p_0 p_1}$$

(a) Assuming that the poles are conjugate symmetric, show that the denominator polynomial has real-valued coefficients. 6 points.

(b) Design the transfer function for a second-order analog filter by determining the locations of the two poles and two zeros and determining the scaling constant  $b_0$ . The filter is to pass frequencies between -10 Hz and 10 Hz and zero out (notch out) frequencies at -60 Hz and 60 Hz. 18 points.

**Problem 1.5** Potpourri. 16 points.

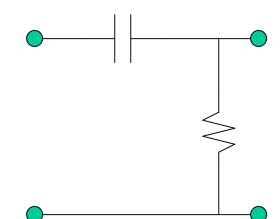
- (a) Either prove the following statement to be true, or give a counterexample to show that the following statement is false: The discrete-time convolution of wo finite duration signals always produces a finite duration result that is longer than either of the signals being convolved. 4 points.

(b) Find the transfer function for the following continuous-time linear time invariant system with input x(t) and output y(t) governed by the differential equation (for  $t \ge 0^-$ ). 4 points.

$$\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = \frac{d}{dt}x(t) + x(t)$$

Be sure to simplify the transfer function as much as possible.

- (c) For the continuous-time analog linear time-invariant circuit on the right,
  - i. Give the transfer function? 4 points.



ii. What kind of filter is it? Lowpass, highpass, bandpass, bandstop, notch, or allpass? 4 points.